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Technical Notes

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Modified Model for Effective Thermal Conductivity of Nanofluids Containing Carbon Nanotubes

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Nomenclature

d	=	diameter of the carbon nanotubes
G	=	(effective) interfacial conductance between carbon nanotubes and liquid, reciprocal of interfacial resistance
k_b	=	thermal conductivity of the base fluid
k_{eff}	=	effective thermal conductivity of the created nanofluid
k_p	=	thermal conductivity of the carbon nanotubes
L	=	length of the carbon nanotubes
η	=	average length efficiency
ϕ	=	real volume fraction of the carbon nanotubes

I. Introduction

NANOFLUIDS [1], a class of fluids with nanosolid particles dispersed, have attracted great interest recently from both experimental [2–11] and theoretical [12–19] investigations, because of their enhanced thermal conductivity (TC) compared with traditional heat transfer fluids like water, ethylene glycol, decene, and engine oil. Nanosolid particles suspended in fluid can be many kinds of materials (such as metal, metal oxide, carbon nanotubes (CNTs), and so on) that have much higher TC than the base fluid [1]. Among the materials applied, the CNT is the most interesting candidate, for its TC is even much larger than the TC of metals. There are three types of CNTs in the application, which are single-walled CNTs (SWCNTs), double-walled CNTs (DWCNTs), and multi-walled CNTs (MWCNTs).

Figure 1 shows the summary of experimental data of TC enhancement of nanofluids containing CNTs that appeared in the literature. It can be seen from Fig. 1 and related literature that the relation between TC enhancement and volume fraction of CNTs would be strongly nonlinear (upward [4,5,9] or downward [2,11] concave), which is quite different from the corresponding linear relation for nanofluids with other non-high-aspect-ratio nanoparticles [1]. Here, we would like to point out that the authors of experimental work in the literature normally discussed their technique and accuracy of measurement before application to nanofluids in their papers. Based on their descriptions, experimental errors of data in Fig. 1 are below 5%. Therefore, we believe that experimental errors do not change the curve shapes in Fig. 1, which are strongly nonlinear.

Several techniques were applied to measure the TC of nanofluids, and most works were done at room temperature. The key issue in the measurement is to avoid convection phenomenon when the TC of fluid is measured. The extensively used technique is the transient hot-wire technique [3,4], and other applied techniques include the transient-plane-source technique [6,7], the steady-state parallel-plate method, and so on.

From a theoretical view, many analytical models [12–19] were developed to predict the effective TC of nanofluids containing CNTs. Based on our simulation, only Yamada and Ota's model [13] obtains upward concave relation between TC enhancement and the volume fraction of CNTs, whereas others obtain an almost linear curve.[‡] In our simulation, $k_p = 2000$ W/m · K, $k_b = 0.15$ W/m · K, $d =$ 25 nm, $L = 20$ μm, and $G = 1.2 \times 10^7$ W/m^2 · K [20].

Through comparing theoretical predictions and experimental results, we see that no one existing model can explain all experimental data satisfactorily. The aim of this work is to develop some expression for effective TC of nanofluids containing CNTs, which can explain or predict almost all experimental data. The idea and simulation results will be described in Secs. II and III.

II. New Analytical Model

Eastman et al [1] summarized several main mechanisms to explain the heat transfer in nanofluids with nanoparticles dispersion: 1) Brownian motion of nanoparticles, 2) heat transfer in nanoparticles, 3) layering of liquid molecules at the liquid/solid interface, 4) interfacial thermal resistance at the liquid/solid interface, and 5) nanoparticle clustering. Let us discuss these mechanisms when nanoparticles are CNTs. The contribution of Brownian motion can be negligible for the high aspect ratio of CNTs. It is well known that heat transfer in CNTs can be simplified as one-directional heat transfer through the alignment axis. Because the types of components (CNTs, base fluid, and surfactant) and their relative weight ratios in nanofluids would be different in various experimental and application situations, it will be almost impossible to estimate the TC of liquid layering, thickness of liquid layering, interfacial resistance at solid/liquid interface, and their influence on complete, effective TC of nanofluids. To simplify the formula used in the application, we prefer to combine the possible effects of liquid layering and interfacial resistance into one effective interfacial resistance. To minimize the enhancement of viscosity and keep fluids stable, we do not think that a large cluster is a good practical idea in the application of nanofluids. Because of entanglement of CNTs, the effective length of CNTs to contribute to heat transfer will not be the same as the real length of CNTs [16]. Therefore, to predict the TC of nanofluids containing CNTs, we should include the effect of interfacial resistance and the effect of entanglement in the model.

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‡We believe that in [12], the authors used the estimated value $L_x = 0.4986$ in their computation for curves in their Figs. 4–5. If one uses exact L_x (e.g., 0.4999) calculated from Eq. 7 in [12], then the nonlinear performance of curves in Fig. 4 of [12] will be gone.)

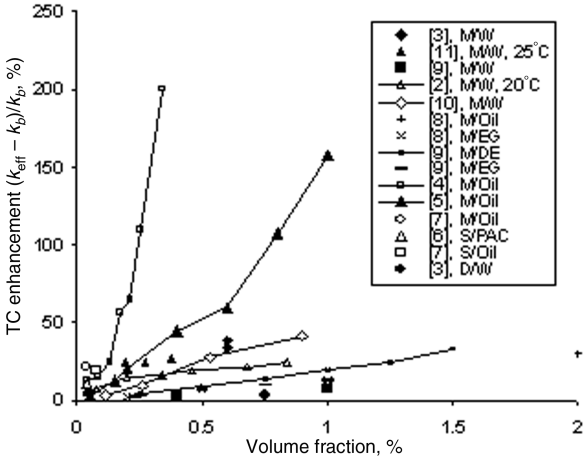

Fig. 1 TC enhancement of nanofluids containing CNTs from experimental work (M means MWCNTs, D means DWCNTs, S means SWCNTs, W means water, PAC means Prestone antifreeze/coolant, EG means ethylene glycol, and DE means decene).

Based on the preceding discussion, we develop a new analytical model for effective TC of nanofluids containing CNTs. The basic idea is to introduce (effective) interfacial resistance [14,16] and average length efficiency (ALE) [16] (used to denote the strength of entanglement) into the original Yamada and Ota model [13], and the ALE is served as an empirical factor. This procedure is quite similar to the modification [16] on the Nan et al. model [14]. The differences between this work and [16] are as follows:

1) The models to be modified are not the same. Nan's [14] model predicts the linear relation between TC enhancement and volume fraction of CNTs, whereas Yamada and Ota's [13] model predicts upward concave relation.

2) Nanocomposites, which are solid, are discussed in [16], whereas nanofluids, which are fluids, are discussed in our work. The CNTs in nanocomposites are not able to move, whereas the CNTs in nanofluids are moveable. It can be expected that the dependence of the ALE on volume fraction and other conditions (in creating and storing procedures) in nanocomposites would be much weaker than the corresponding dependence in nanofluids. In fact, we simulated the model in [16] and got a linear relation between TC enhancement and volume fraction of CNTs, which does not explain the upward concave shape in the experimental work.

After some simple implement, the final formulas of our model are as follows:

$$\frac{k_{\text{eff}}}{k_b} = \frac{(k_{p,m}/k_b) + \alpha - \alpha\phi_N[1 - (k_{p,m}/k_b)]}{(k_{p,m}/k_b) + \alpha + \phi_N[1 - (k_{p,m}/k_b)]} \tag{1}$$

where

$$k_{p,m} = \frac{k_{p,N}}{1 + (2k_{p,N}/L_N G)}, \qquad \alpha = 2\phi_N^{0.2}\frac{L_N}{d}, \qquad \phi_N = \eta\phi$$
$$L_N = \eta L, \qquad k_{p,N} = \eta k_p \tag{2}$$

If one changes $k_{p,m} \to k_p$, $\phi_N \to \phi$, and $\alpha = 2\phi^{0.2}L/d$ in Eq. (1), the original Yamada and Ota model [13] will be recovered.

III. Results and Discussion

ALE is a statistical quantity, which means that the length distribution function of CNTs (as a function of volume fraction, type of CNTs, type of base fluid, etc.) needs to be determined to give its value accurately. Therefore, it is almost impossible to obtain the accurate value of the ALE. In the application, we would like to use this quantity as an empirical factor, which needs to satisfy some rules, such as its value should locate between zero and one, and these values at the same experimental situation (such as an experiment from the same research group) should be similar or decrease when the volume fraction increases. Because the ALE concept is quite new and it is beyond the scope of this Technical Note to do systematic study, what we consider here is the feasibility of our new model. The way we used it in the test is to calculate the ALE from experimental data of the TC of nanofluids and to further check whether these values are reasonable (e.g., satisfy the two rules mentioned earlier). If these values make sense, then the feasibility of the new model is justified.

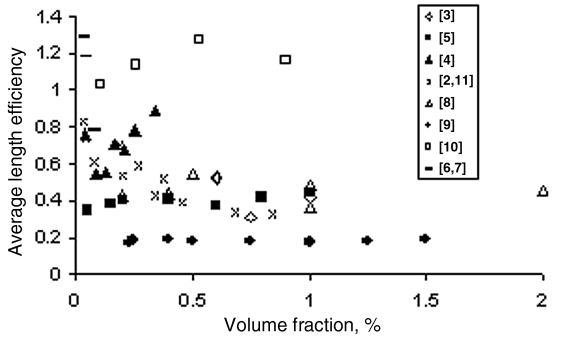

Fig. 2 ALE calculated from our model with experimental data of the TC of nanofluids.

Figure 2 shows the ALEs calculated from our model with experimental data of the effective TC of nanofluids. In the calculation of this part, the TC of MWCNTs, DWCNTs, and SWCNTs are chosen as 2000.0, 4000.0, and 6000.0 $\text{W/m} \cdot \text{K}$, $G = 1.2 \times 10^7\ \text{W/m}^2 \cdot \text{K}$, and other quantities are determined from individual literature. It can be seen from Fig. 2 that the preceding two issues of ALE are indeed satisfied for almost all experimental data, and this point shows the feasibility of our model.

IV. Conclusions

Nanofluids, fluids dispersing nanoparticles (metal, oxide, carbon nanotubes, etc.), were demonstrated to enhance the TC of fluids, and so these fluids can be applied for high heat-flux applications such as superfast computing, etc. The experimental works show that the relation between TC enhancement and volume fraction of CNTs would be strongly nonlinear (upward or downward concave), which cannot be explained by any single existing analytical model. In this work, we develop a new model for the TC of nanofluids. The basic idea is to introduce interfacial resistance between CNTs and liquid and the ALE of CNTs into the original Yamada and Ota [13] model, and the average length efficiency is served as an empirical factor. The feasibility of this model is also discussed in this work. Future work will cover how to obtain the ALE and the suggested values of the ALE in various application situations.

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